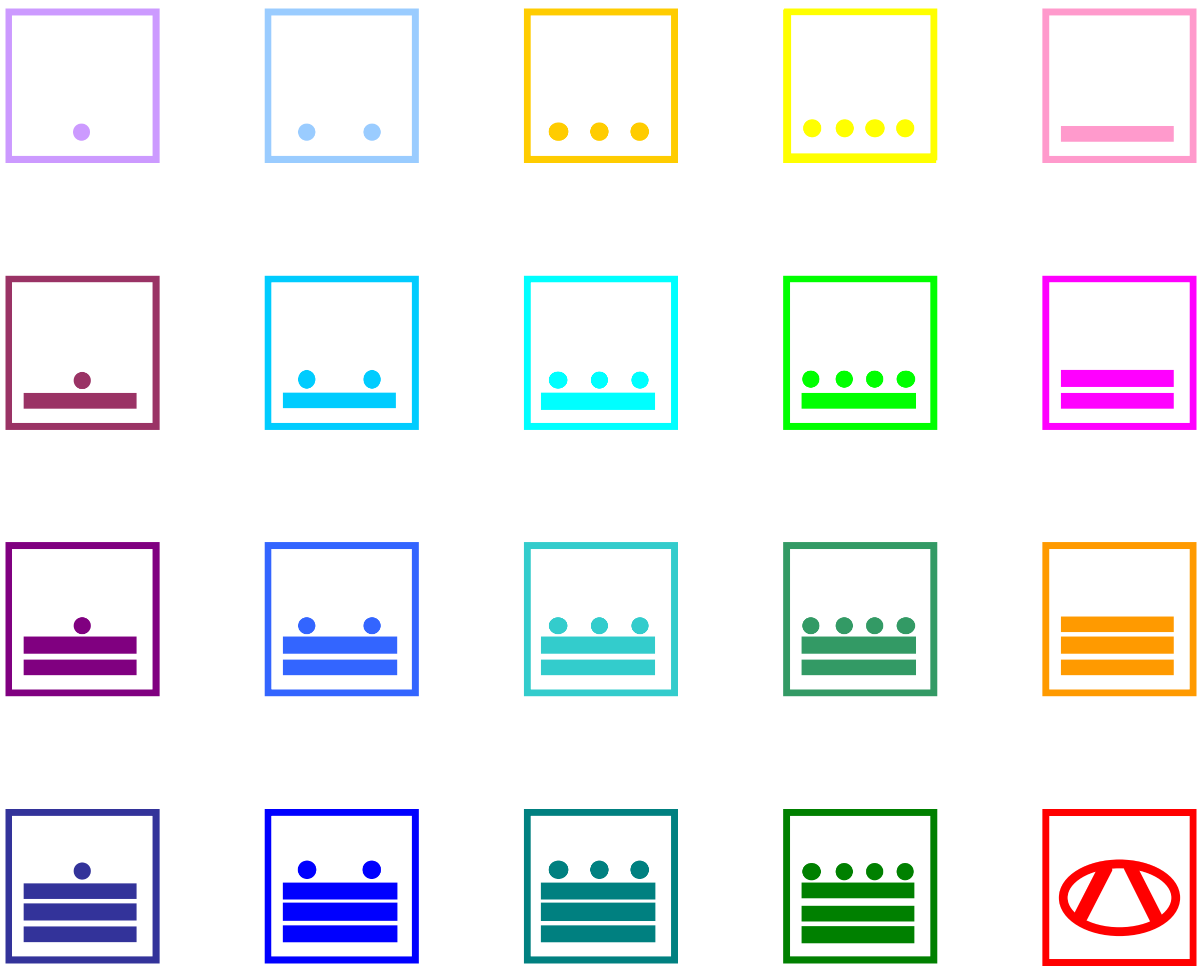
How to calculate with

Mayan Numbers



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***nitac14b@yahoo.com*.**

Why Did I Write This?

Two years ago, my wife took a course in Tzotzil (a Mayan dialect) in San Cristóbal's *Universidad Maya*. One day she told me that when the professor was teaching them about Mayan numbers, the students had asked him, "What are they good for? For example, can you add and subtract with them?"

I've done a lot of volunteer tutoring in math, and I've found that students do better when they understand the properties of numbers—for example, the commutative, the distributive, and the associative. For some students, a light comes on when I then proceed to show that our procedures for adding, subtracting, multiplying, and dividing are nothing more than convenient ways to take advantage of those properties. Perhaps these experiences explain why the Tzotzil students' question made me think, "Our procedures for adding, etc., should work with Mayan numbers, too. After all, Mayan numbers have a place-value system and a symbol for zero just like our Hindu-Arabic numbers. The fact that Mayan numbers are base 20 rather than base 10 shouldn't make any difference."

So I tried a few calculations with Mayan numerals, and it turned out that the same procedures did indeed work. I went on to make this collection of calculations for my wife's professor and classmates. Included are an addition, a subtraction, two multiplications, a division, and—as something really different—a square root. I hope this will be sufficient; if you want to see how to do cube roots or logarithms with Mayan numerals, you'll have to look elsewhere! (Or do them yourself, using the ideas presented here).

The author

San Cristóbal de Las Casas, Chiapas, Mexico
11 February 2005

P.S. For typographic reasons, this document follows the convention of using a comma to separate the units' place from the tenths' place, and a period to separate the hundreds' place from the thousands', etc.

P.P.S. I really must make a confession. I did all of these calculations with Mayan numerals, and only then checked the results by doing the same calculations with Hindu-Arabic numerals. To my embarrassment, there were four or five times when I did the calculation correctly with Mayan numerals, then messed up when I "checked" the results….

How to Calculate with Mayan Numbers

Mayan numbers aren't just a curiosity—they're a completely practical tool for doing calculations. Just like Hindu-Arabic (HA) numbers, Mayan numbers have a place-value system, a consistent base (20 instead of 10), and a placeholder symbol, analogous to our zero, to show when a place is empty.

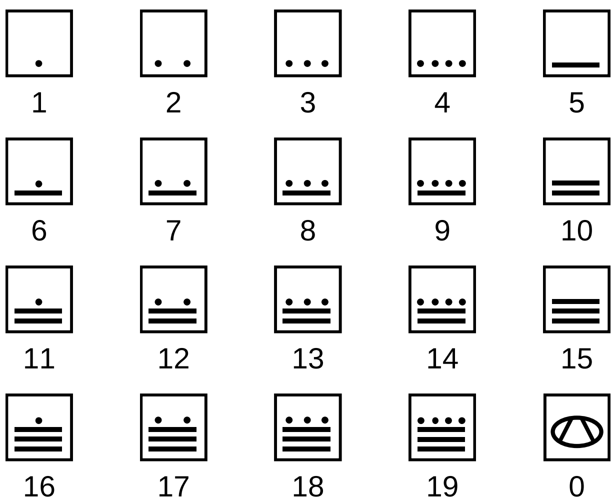
Thanks to these features of the Mayan number system, we can add, subtract, multiply, divide, and even find square roots using the same techniques that work with HA numerals. (For example, borrowing and carrying digits). Of course, it's necessary to carry out these operations according to Mayan addition and multiplication tables.

We'll begin by presenting the Mayan digits and an example of how to write a multiple-digit numeral. Then, we'll see how to do the above-mentioned calculations.

Mayan Digits, and How to Write Multiple-Digit Numbers

◆ Mayan Digits

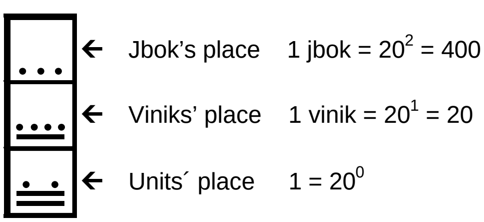
Here are the Mayan digits. Unlike HA digits, the bigger the number represented, the more space the digit occupies (except in the case of zero).



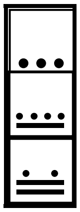
Please note that the attached addition and multiplication tables use symbol ✿ instead of .

◆ Multiple-Digit Numbers

In multiple-digit numbers, place values increase vertically. *Jbok'* and *vinik* are Tzotzil words.



So, what number have we just finished reading?



3 × 400 = 1.200

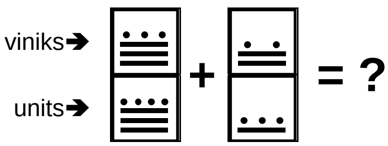
9 × 20 = 180

12 × 1 = 12

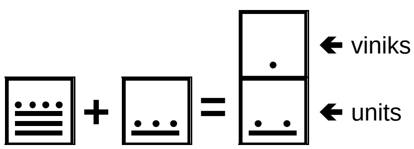
1.200 + 180 + 12 = 1.392

Calculations with Mayan Numbers

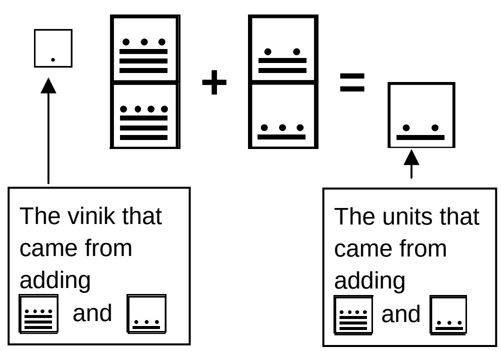
◆ An addition



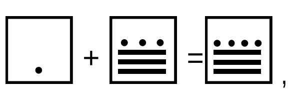
As is the case for HA numbers, we begin by summing the digits in the units' place. According to the addition table,



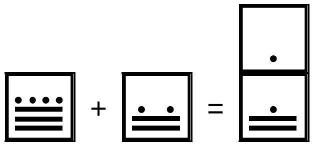
Therefore, the sum of the units has as its digit in the units' place, and in the viniks' place. Just as with HA numbers, we write the units in the units' place, and carry the vinik to the viniks' place:



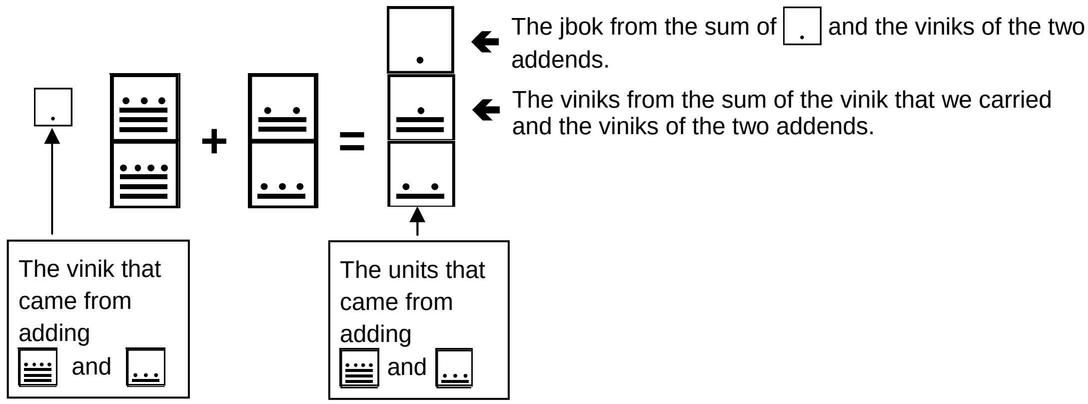
Now, we add the viniks. First, we add the that we carried from the sum of the units, to the that was already in the viniks' place of the first addend. According to the addition table,



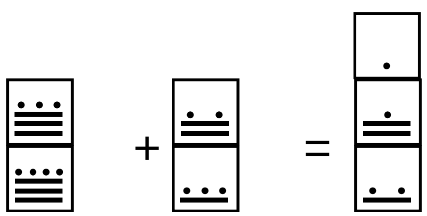
and we now we this result to the digit that was already in the viniks' place of the second addend:



We write the in the viniks' place of the answer, and carry the to the jbok's place. Since the addends have no digits in the jbok's place, we just write the digit in the jbok's place of the answer:



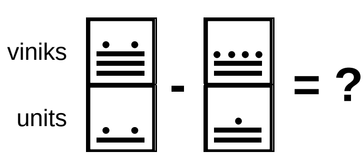
Note that we've finished adding the two numbers, thereby coming up with a third that is presumably the right answer, and we still don't know what any of these numbers are! Actually, this shouldn't surprise us. When we add multiple-digit HA numbers, we do so by manipulating symbols according to rules that are based upon properties of real numbers. (Especially the associative property of addition.) Therefore, it's not necessary to know what numbers the symbols represent. If we do things that are permitted by the properties of real numbers, we get valid results. All this having been said, however, we really should check our answer.



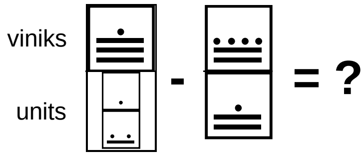
18x20 = 360	12x20 = 240	1x400 = 400
19x1 = 19	8x1 = 8	11x20 = 220
Total: 379	Total: 248	7x1 = 7
		Total: 627

379 + 248 = 627. OK.

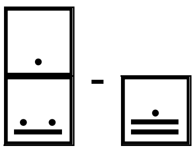
◆ A Subtraction



We begin with the units. Since the units of the minuend (*i.e.*, the number from which we are subtracting) are less than the units of the subtrahend (the number being subtracted), we have to borrow from the viniks, just as is the case with HA numbers. After borrowing, the problem looks like this:

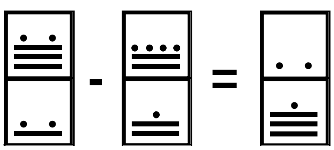


Now, in the units´ place we have the subtraction



How shall we find the answer? By looking for the result in the column of in the addition table. Upon finding it, we note that it is also in the row of . Therefore, we write this digit in the units´ place in the answer.

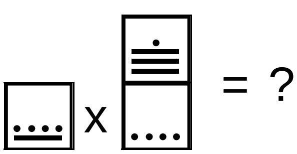
Proceeding now to the viniks' place, we have the subtraction - , and looking for in the column of , we find that it is also in the row of . Therefore, - = . We write this digit in the viniks' place of the answer. The result is



347 - 291 = 56. OK.

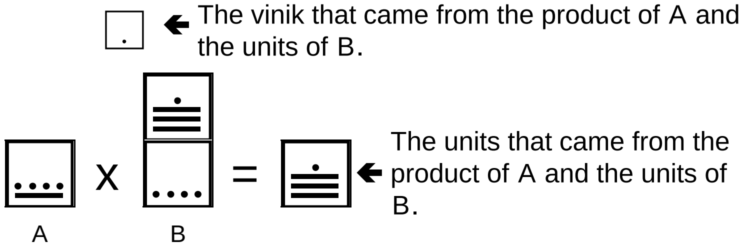
◆ Two multiplications

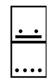
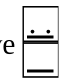
First we'll do a multiplication with a one-digit multiplier to demonstrate the technique. We'll then do a multiplication with multiple-digit numbers.

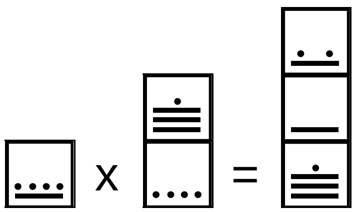


A B

First we multiply the units of B by A. Again, the process is identical to that used with HA numbers. Looking in the multiplication table, we find that x = . Therefore, we write in the units' place of the answer, and we write to one side, to be added later to the product of A and the viniks of B.

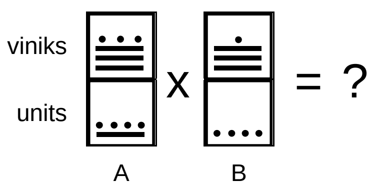


Now we multiply the viniks of B by A:  x  =  viniks. Adding this result to the vinik that we'd already written to one side, we have  viniks, so that the complete result is

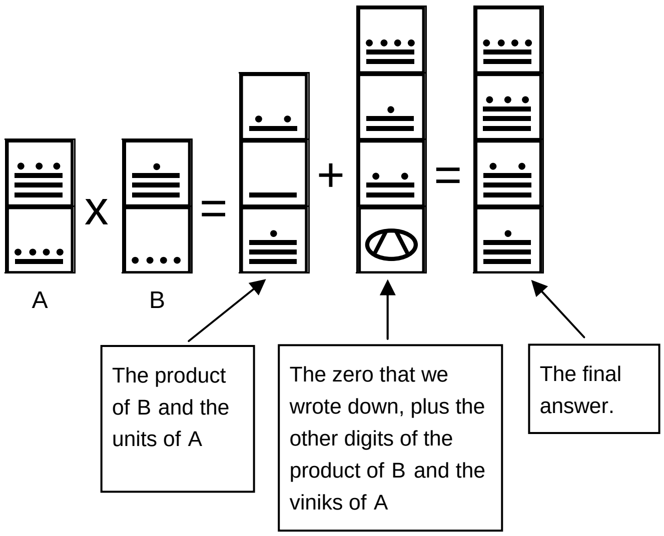


9 x 324 = 2.916 OK.

As the second multiplication, we'll add to A a digit in the viniks' place:



First we multiply B by the units of A; we already have that result from the previous multiplication. Then, we multiply B by the viniks of A. There are two steps in said multiplication. First, as is the case with HA numbers, we write a "zero" in the units' place. Next, we find the other digits by the technique that we just finished learning. That is, the technique by which we found the product of B and the units of A. When we're finished with these two multiplications, we add the products. The result is

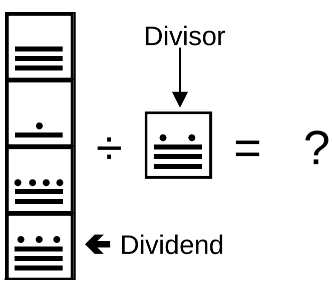


369 x 324 = 116.640 + 2.916 = 119.556. OK.

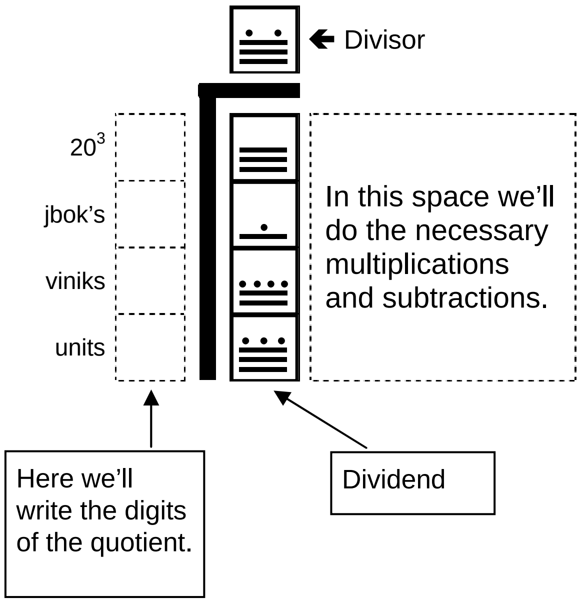
It's worth mentioning three things. First, we did all of this without knowing what numbers are represented by A and B. Second, we did not "multiply" the two numbers in the sense that we multiply 3 and 4. Instead, we followed a procedure that gave their product. We didn't know what number this product represented, either. Third, I claim that this is often the case when we multiply HA numbers, too. That is, when we multiply 3 and 4, for example, we can think "3 four's", and visualize that the answer is 12. However, few people can find the product of 3.117.235 and 1.259.003 in the same way! Moreover, the answer (3.924.608.216.705) is a number that we really don't know. We know what each of its digits represents, of course, and we may know how to name this number, but who really understands the quantity that this number represents?

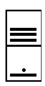
◆ A division

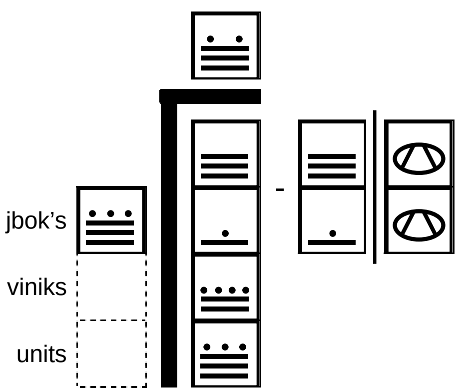
As is the case with multiplication, "division" is usually a multi-step process rather than a single operation.



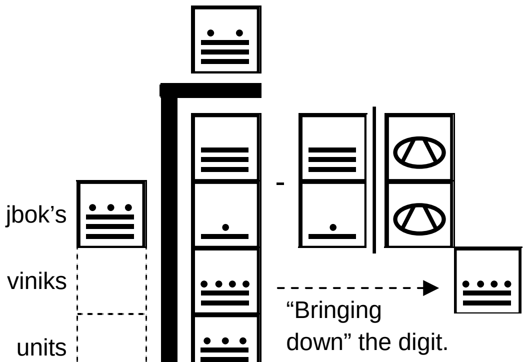
Let's write this problem in a form that's more convenient to solve:



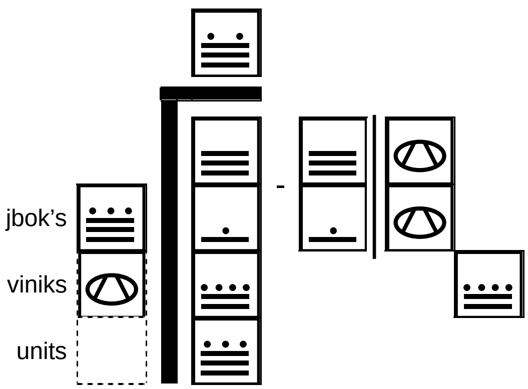
As is the case with HA numbers, division is in part a process of trial and error. First, just as we do with HA numbers, we ask whether the divisor goes into the first digit of the dividend. It doesn't, so we don't write anything in the 20^3 place of the quotient. Now, we ask whether the divisor goes into the first *two* digits of the dividend (specifically, ). Looking in the  column of the multiplication table, we see that  is the product of  and . Therefore, we write this in the jbok's place of the quotient. Next, we do the corresponding multiplication and subtraction:



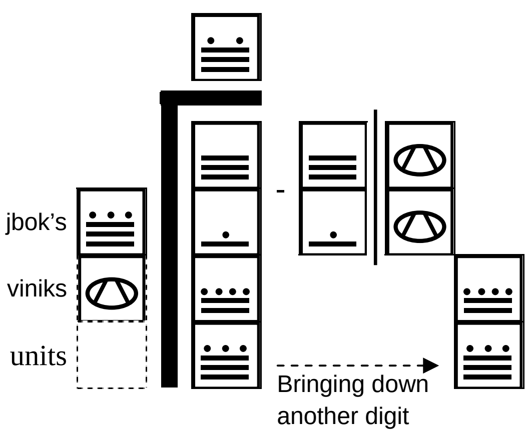
Now we "bring down" the viniks' digit of the dividend.

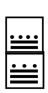
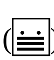
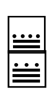
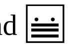


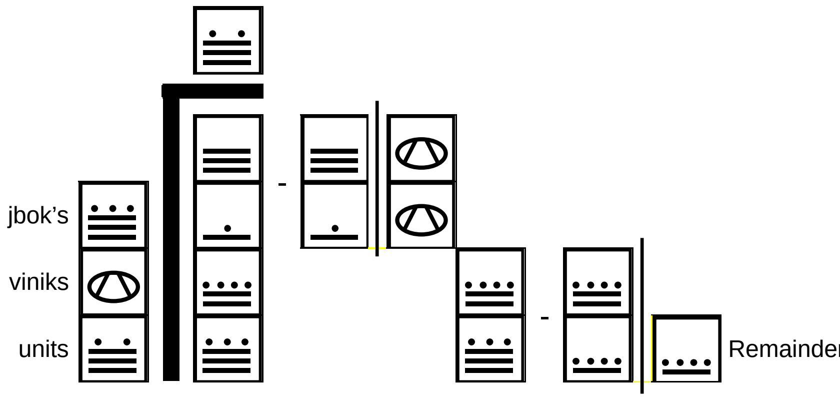
Since the divisor doesn't go into this digit, we write a zero in the viniks' place of the quotient.



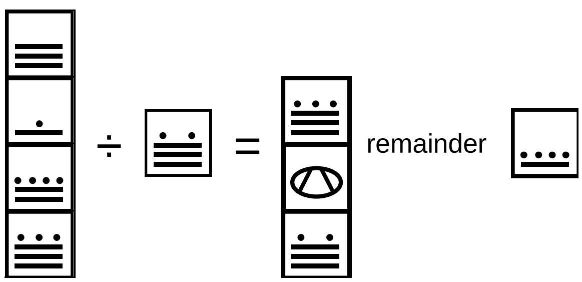
Next, we bring down the units' digit of the dividend.



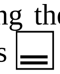
Does the divisor go into  ? Certainly, but how many times? Searching the multiplication table, in the column of the divisor (), we find that  is smaller than the product of the divisor and  , and larger than the product of the divisor and  . Therefore, we write  in the units' place of the quotient. We also do the corresponding multiplication and subtraction:

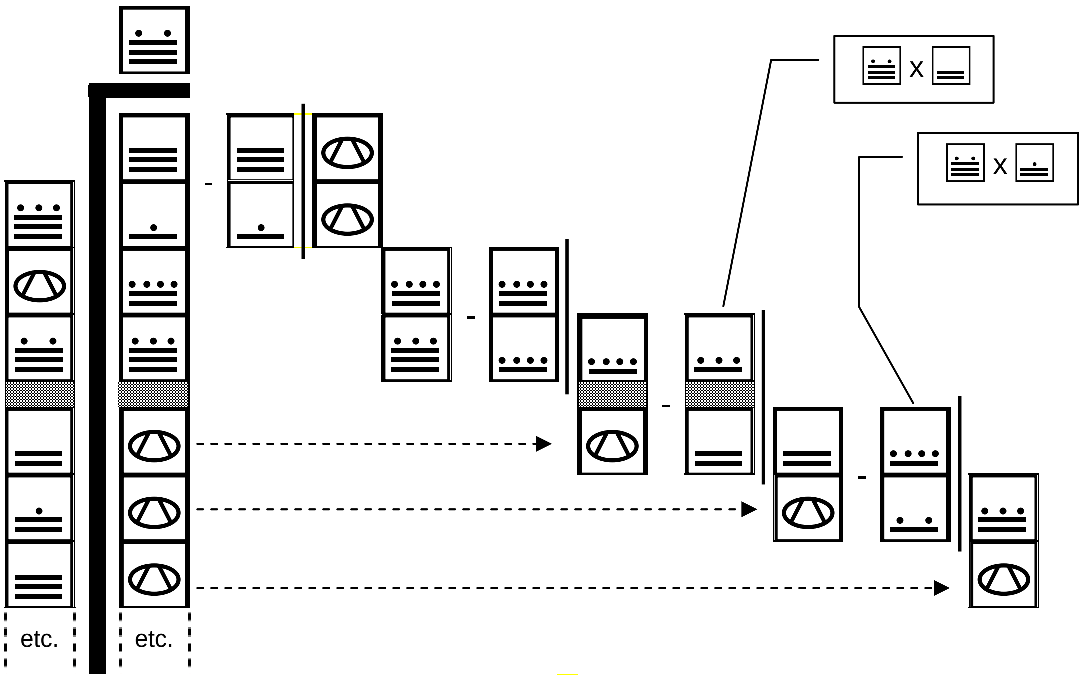


Now we're finished, and the answer is

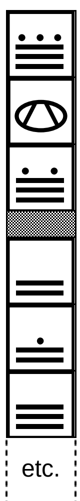


122.698 ÷ 17 = 7.217 remainder 9. OK.

Note that it's not necessary to settle for an answer in the form of a quotient and a remainder. Instead, we can obtain a solution with a "decimal comma" and additional digits in much the same way as we obtain a HA-number solution with digits to the right of the decimal comma. All that's necessary is to write "zeros" below the units' place in the dividend, and "bring them down" as necessary. In this way, we can find that the first digit below the decimal comma is  Here is the work needed to obtain a quotient with three "decimal places", the  being the decimal comma:



Note the place values below the decimal comma. Each is $\frac{1}{20}$ of the value of the place above it.

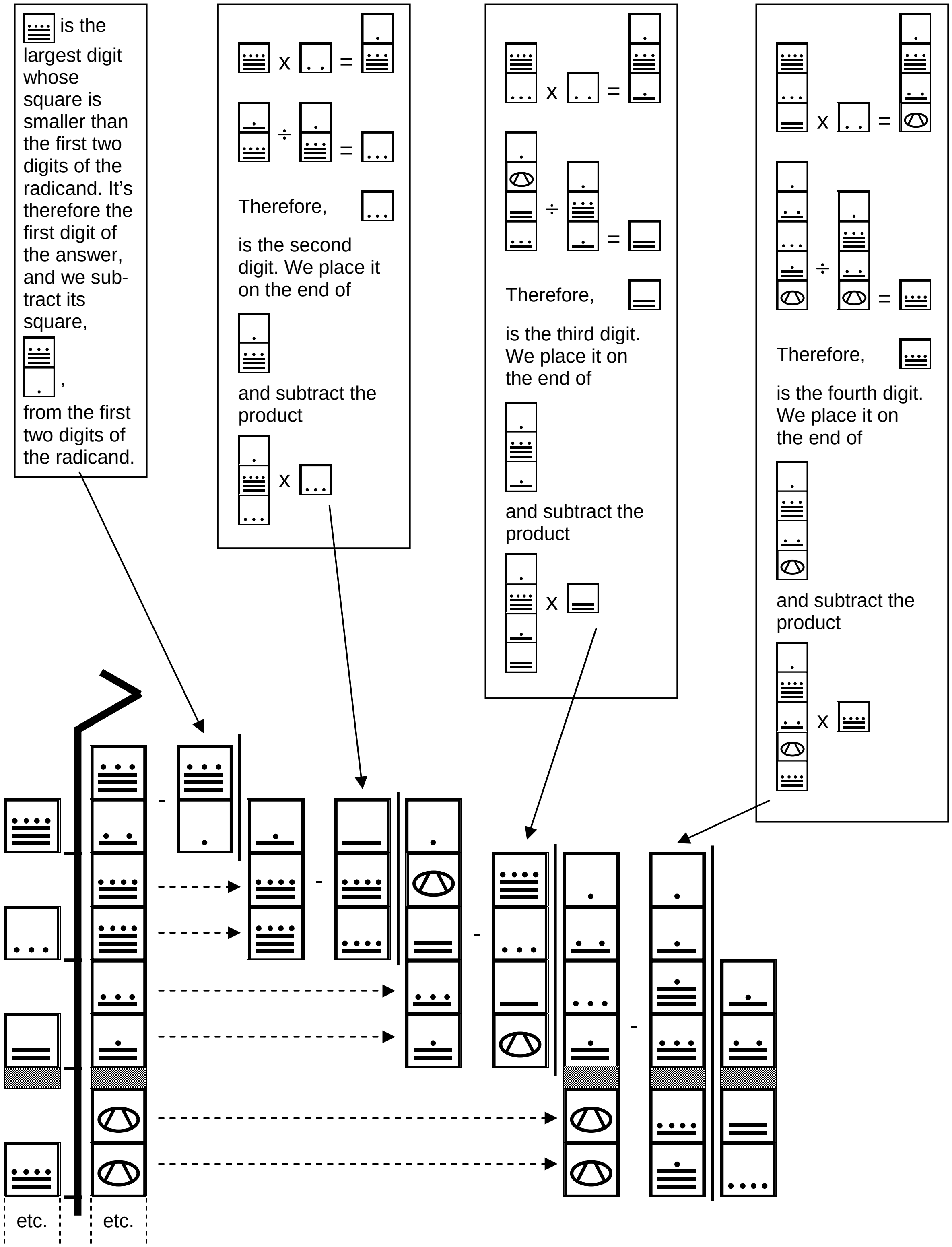


400	18 x 400 =	7.200
20	0 x 20 =	0
1	17 x 1 =	17
$\frac{1}{20}$	10 x $\frac{1}{20}$ =	0,5
$\frac{1}{400}$	11 x $\frac{1}{400}$ =	0,0275
$\frac{1}{8000}$	15 x $\frac{1}{8000}$ =	0,001875

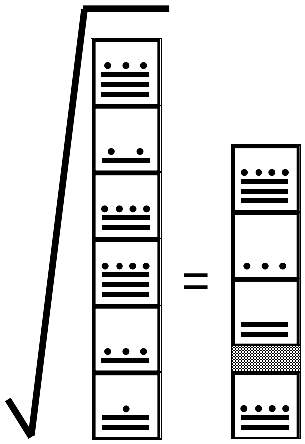
Just as the representation of $\frac{1}{3}$ in HA numbers never terminates because the denominator (3) is not a factor of 10, the representation of $7217\frac{9}{17}$ in Mayan numbers will never terminate because 17 is not a factor of 20 (the base of the Mayan number system). I don't know whether the Mayans did calculations with decimal numbers, but their marvelous number system offered them this possibility.

◆ Extraction of a square root

The algorithm is the same as used with HA numbers. Since it uses operations that we've already learned (*i.e.*, multiplications, divisions, and subtractions), I've omitted the details.



The answer is



$\sqrt{58.839.771}$ = 7670,7 to one decimal place. OK.

I doubt that the Mayans ever went to the trouble of inventing this procedure because there are less-complicated ones that work perfectly well. However, their number system would have allowed them to use it.

Summary

It's clear that the Mayan number system allows us to add, subtract, multiply, and divide with the same techniques as used for HA numbers. This is possible because both systems have a consistent base, a place-value system, and a symbol to show when a place is empty. Therefore, if I'm not mistaken, any calculation possible with HA numbers can be done with Mayan numbers, using the same algorithms.

Addition and Multiplication Tables

+	𝋡	𝋢	𝋣	𝋤	𝋥	𝋦	𝋧	𝋨	𝋩	𝋪	𝋫	𝋬	𝋭	𝋮	𝋯	𝋰	𝋱	𝋲	𝋳	𝋡𝋠	+
𝋡	𝋢	𝋣	𝋤	𝋥	𝋦	𝋧	𝋨	𝋩	𝋪	𝋫	𝋬	𝋭	𝋮	𝋯	𝋰	𝋱	𝋲	𝋳	𝋡𝋠	𝋡𝋡	𝋡
𝋢	𝋣	𝋤	𝋥	𝋦	𝋧	𝋨	𝋩	𝋪	𝋫	𝋬	𝋭	𝋮	𝋯	𝋰	𝋱	𝋲	𝋳	𝋡𝋠	𝋡𝋡	𝋡𝋢	𝋢
𝋣	𝋤	𝋥	𝋦	𝋧	𝋨	𝋩	𝋪	𝋫	𝋬	𝋭	𝋮	𝋯	𝋰	𝋱	𝋲	𝋳	𝋡𝋠	𝋡𝋡	𝋡𝋢	𝋡𝋣	𝋣
𝋤	𝋥	𝋦	𝋧	𝋨	𝋩	𝋪	𝋫	𝋬	𝋭	𝋮	𝋯	𝋰	𝋱	𝋲	𝋳	𝋡𝋠	𝋡𝋡	𝋡𝋢	𝋡𝋣	𝋡𝋤	𝋤
𝋥	𝋦	𝋧	𝋨	𝋩	𝋪	𝋫	𝋬	𝋭	𝋮	𝋯	𝋰	𝋱	𝋲	𝋳	𝋡𝋠	𝋡𝋡	𝋡𝋢	𝋡𝋣	𝋡𝋤	𝋡𝋥	𝋥
𝋦	𝋧	𝋨	𝋩	𝋪	𝋫	𝋬	𝋭	𝋮	𝋯	𝋰	𝋱	𝋲	𝋳	𝋡𝋠	𝋡𝋡	𝋡𝋢	𝋡𝋣	𝋡𝋤	𝋡𝋥	𝋡𝋦	𝋦
𝋧	𝋨	𝋩	𝋪	𝋫	𝋬	𝋭	𝋮	𝋯	𝋰	𝋱	𝋲	𝋳	𝋡𝋠	𝋡𝋡	𝋡𝋢	𝋡𝋣	𝋡𝋤	𝋡𝋥	𝋡𝋦	𝋡𝋧	𝋧
𝋨	𝋩	𝋪	𝋫	𝋬	𝋭	𝋮	𝋯	𝋰	𝋱	𝋲	𝋳	𝋡𝋠	𝋡𝋡	𝋡𝋢	𝋡𝋣	𝋡𝋤	𝋡𝋥	𝋡𝋦	𝋡𝋧	𝋡𝋨	𝋨
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𝋫	𝋬	𝋭	𝋮	𝋯	𝋰	𝋱	𝋲	𝋳	𝋡𝋠	𝋡𝋡	𝋡𝋢	𝋡𝋣	𝋡𝋤	𝋡𝋥	𝋡𝋦	𝋡𝋧	𝋡𝋨	𝋡𝋩	𝋡𝋪	𝋡𝋫	𝋫
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𝋰	𝋱	𝋲	𝋳	𝋡𝋠	𝋡𝋡	𝋡𝋢	𝋡𝋣	𝋡𝋤	𝋡𝋥	𝋡𝋦	𝋡𝋧	𝋡𝋨	𝋡𝋩	𝋡𝋪	𝋡𝋫	𝋡𝋬	𝋡𝋭	𝋡𝋮	𝋡𝋯	𝋡𝋰	𝋰
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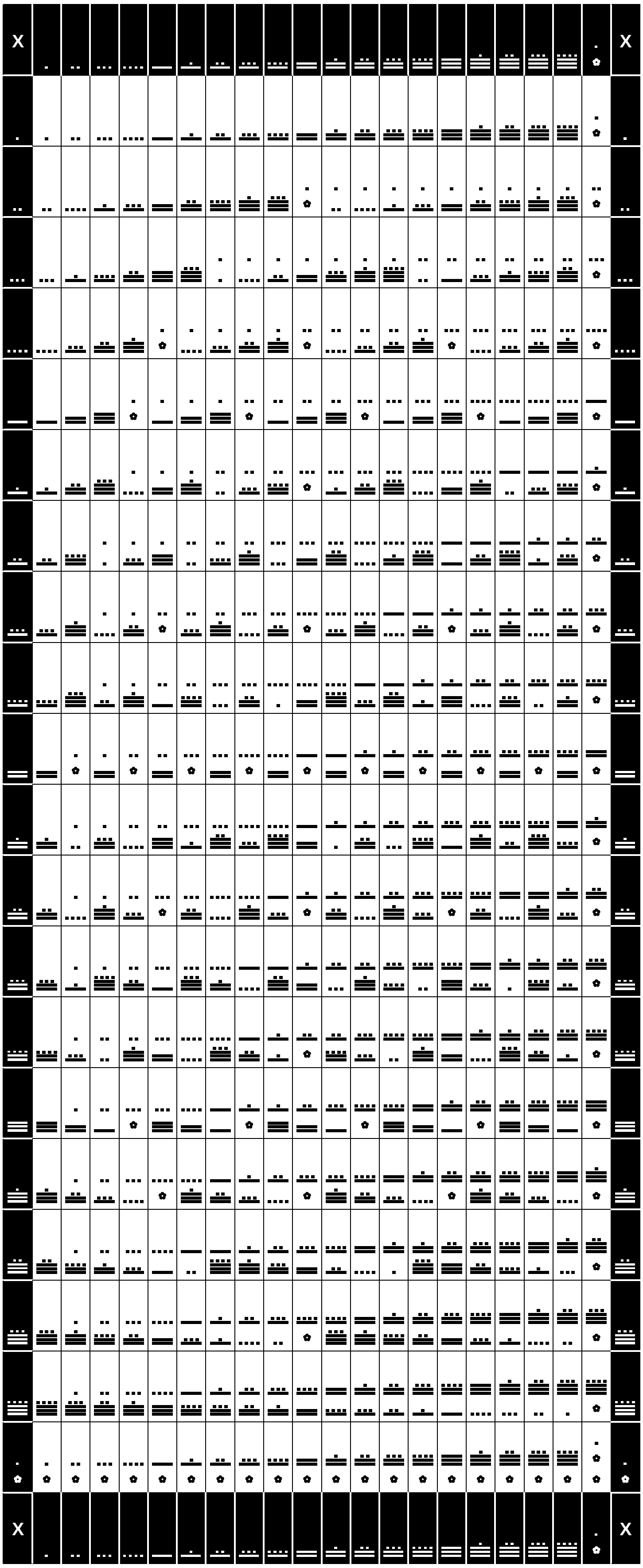
+	𝋡	𝋢	𝋣	𝋤	𝋥	𝋦	𝋧	𝋨	𝋩	𝋪	𝋫	𝋬	𝋭	𝋮	𝋯	𝋰	𝋱	𝋲	𝋳	𝋡𝋠	+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